Sol. to Quiz 3. Cal. BM Yes

1. (a) $\int \frac{x^2+3}{(x^3+9x-4)^2}\,dx$ 令 $u = x^3+9x-4$ $\Rightarrow du = (3x^2+9)\,dx$ $\Rightarrow (x^2+3)\,dx = \frac{1}{3}du$

$= \frac{1}{3}\int \frac{1}{u^2}\,du = \frac{1}{3}\int u^{-2}\,du$

$= -\frac{1}{3}u^{-1} + C = -\frac{1}{3}\frac{1}{x^3+9x-4} + C$

(b) $\int \frac{x^2+2x+5}{x-1}\,dx$

$$\begin{array}{r} x+3 \\ x-1\,\overline{)\,x^2+2x+5} \\ \underline{x^2-x} \\ 3x+5 \\ \underline{3x-3} \\ 8 \end{array}$$

$= \int x+3+\frac{8}{x-1}\,dx$

$= \frac{1}{2}x^2+3x+8\ln(|x-1|)+C$

$\because \int \frac{1}{x-1}\,dx \overset{u=x-1}{=} \int \frac{1}{u}\,du = \ln(|u|)+C = \ln(|x-1|)+C.$

(c) $\int \frac{1}{x\ln x}\,dx$ 令 $u=\ln x \Rightarrow du = \frac{1}{x}dx$

$= \int \frac{1}{u}\,du = \ln(|u|)+C = \ln(|\ln x|)+C.$

(d) $\int \frac{1+e^{-x}}{1+xe^{-x}}\,dx$

$= \int \frac{e^x}{e^x}\,\frac{1+e^{-x}}{1+xe^{-x}}\,dx = \int \frac{e^x+1}{e^x+x}\,dx$

$= \int \frac{1}{u}\,du$ 令 $u = e^x+x$ $\Rightarrow du = (e^x+1)\,dx$

$= \ln(|u|)+C = \ln(|e^x+x|)+C.$

(e) $\int \frac{e^{2x}+2e^x+1}{e^x}\,dx$

$= \int e^x\,dx + \int 2\,dx + \int e^{-x}\,dx$

$= e^x+2x-e^{-x}+C.$ $\because \int e^{-x}\,dx \overset{u=-x}{=} -\int e^u\,du$

a). (f) $\int x^2(\ln x)^3\,dx$

$= \int \frac{1}{3}(\ln x)^3\,dx^3$

$= \frac{x^3}{3}(\ln x)^3 - \int x^3\cdot\frac{1}{3}\cdot 3(\ln x)^2\cdot\frac{1}{x}\,dx$

$= \frac{x^3}{3}(\ln x)^3 - \int x^2(\ln x)^2\,dx$

$= \frac{x^3}{3}(\ln x)^3 - A$

w/ $A = \int x^2(\ln x)^2\,dx = \int \frac{1}{3}(\ln x)^2\,dx^3$

$= \frac{x^3}{3}(\ln x)^2 - \int x^3\cdot\frac{1}{3}\cdot 2(\ln x)\cdot\frac{1}{x}\,dx$

$= \frac{x^3}{3}(\ln x)^2 - \int \frac{2}{3}x^2\ln x\,dx$

$= \frac{x^3}{3}(\ln x)^2 - \frac{2}{3}B$

w/ $B = \int x^2\ln x\,dx = \int \frac{1}{3}\ln x\,dx^3$

$= \frac{x^3}{3}\ln x - \int x^3\cdot\frac{1}{3}\cdot\frac{1}{x}\,dx$

$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx$

$= \frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C$

2. $f(x) = \int \frac{e^{2/x}}{x^2}\,dx$ 令 $u=\frac{2}{x}$, $du = -\frac{2}{x^2}dx$

$= \int -\frac{1}{2}e^u\,du$

$= -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\frac{2}{x}} + C.$

& $f(4) = -\frac{1}{2}e^{\frac{1}{2}} + C = 6 \Rightarrow C = 6+\frac{1}{2}\sqrt{e}$

$\Rightarrow f(x) = -\frac{1}{2}e^{\frac{2}{x}} + 6 + \frac{1}{2}\sqrt{e}.$

3. $f'(x) = \int f''(x)\,dx$

$= \int 2\,dx = 2x+C.$

$\Rightarrow f'(2) = 4+C = 5 \Rightarrow C = 1$

$f'(x) = 2x+1$

$\Rightarrow f(x) = \int f'(x)\,dx = \int 2x+1\,dx$

$= x^2+x+C.$

$\Rightarrow f(2) = 4+2+C = 10 \Rightarrow C = 4$

$\Rightarrow f(x) = x^2+x+4$

4. (a) $\int \frac{\sin x}{1+\cos x}\,dx$ 令 $u = 1+\cos x$, $du = -\sin x\,dx$

$= \int \frac{-1}{u}\,du = -\ln(|u|)+C.$

$= -\ln(1+\cos x)+C.$

(b) $\int \sec^6\frac{x}{4}\tan\frac{x}{4}\,dx$ 令 $u = \tan\frac{x}{4}$ $\Rightarrow du = (\sec^2\frac{x}{4})\frac{1}{4}dx$

$= \int \sec^4\frac{x}{4}\,\sec^2\frac{x}{4}\,\tan\frac{x}{4}\,dx$

$= \int (\tan^2\frac{x}{4}+1)^2\tan\frac{x}{4}\sec^2\frac{x}{4}\,dx$

$= \int (u^2+1)^2\,u\cdot 4\,du$

$= 4\int (u^4+2u^2+1)\,u\,du$

$= 4\int u^5+2u^3+u\,du$

$= \frac{4}{6}u^6 + \frac{4\cdot 2}{4}u^4 + \frac{4\cdot 1}{2}u^2 + C$

$= \frac{2}{3}\tan^6\frac{x}{4} + 2\tan^4\frac{x}{4} + 2\tan^2\frac{x}{4} + C$

new (f)

$\int x(\ln x)^2\,dx = \int \frac{1}{2}(\ln x)^2\,dx^2$

$= \frac{x^2}{2}(\ln x)^2 - \int x^2\ln x\cdot\frac{1}{x}\,dx$

$= \frac{x^2}{2}(\ln x)^2 - \int x\ln x\,dx = \frac{x^2}{2}(\ln x)^2 - \int \frac{1}{2}\ln x\,dx^2$

$= \frac{x^2}{2}(\ln x)^2 - \frac{1}{2}x^2\ln x + \int x^2\cdot\frac{1}{2}\cdot\frac{1}{x}\,dx$

$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{1}{4}x^2 + C$