榮譽第一 Quiz 2. 1/2

國立東華大學
應用數學系____學年度第___學期 考試科目：Cal. BM. □期中□期末考試試卷

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1. $f(x)=\frac{x^2}{x-1} \Rightarrow f'(x)=\frac{(x-1)2x-x^2}{(x-1)^2}$

$=\frac{x^2-2x}{(x-1)^2}=\frac{x(x-2)}{(x-1)^2}=0$

$\Rightarrow x=0,2$ & $f(0)=0$ $f(2)=4$

$\Rightarrow$ At points $(0,0)$, $(2,4)$ f has horizontal tangent line.

2.(a) $f'(t)=(t^2-9)\frac{1}{2}\frac{1}{\sqrt{t+2}}+\sqrt{t+2}\,(2t)$

$\Rightarrow f'(-1)=(1-9)\frac{1}{2}-2=-6$

Tangent line is $y-(-8)=-6(t-(-1))$

i.e. $y=-6t-14$

(b) $f'(x)=\frac{\sqrt{2x-3}\cdot 1-(x+1)\frac{1}{2}\frac{1}{\sqrt{2x-3}}(2)}{2x-3}$

$=\frac{\sqrt{2x-3}-\frac{x+1}{\sqrt{2x-3}}}{2x-3}$

$\Rightarrow f'(2)=\frac{1-3}{1}=-2$

Tangent line: $y-3=-2(x-2)$ i.e. $y=-2x+7$

(c) $\sqrt{xy}=x-2y$ at $(4,1)$

$\frac{d}{dx}(\sqrt{x}\sqrt{y})=\frac{d}{dx}(x-2y)$

$\Leftrightarrow \frac{1}{2\sqrt{x}}\sqrt{y}+\sqrt{x}\frac{1}{2\sqrt{y}}\frac{dy}{dx}=1-2\frac{dy}{dx}$

$\xLeftrightarrow[y=1]{x=4}$ $\frac{1}{4}+\frac{dy}{dx}=1-2\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{4}$

Tangent line: $y-1=\frac{1}{4}(x-4)$, i.e. $y=\frac{x}{4}$

3.(a) $\frac{dy}{dx}=4\tan(4x)\cdot\sec^2(4x)\cdot 4$

$=16\tan(4x)\cdot\sec^2(4x)$

(b) $f'(x)=3(x^3-2x)^2(3x^2-2)$

$f''(x)=6(x^3-2x)(3x^2-2)^2+3(6x)(x^3-2x)^2$

$\Rightarrow f''(1)=6\times(-1)+3\times 6=12$

4.(a) $g'(x)=(x+5)^2+(x+2)2(x+5)$

$=(x+5)(x+5+2x+4)$

$=(x+5)(3x+9)=0$

$\Rightarrow x=-5,-3$: critical #'s.

		-5		-3	
g'	$+$		$-$		$+$
g	$\nearrow$	rel max	$\searrow$	mini	$\nearrow$

i.e. $g(-5)=0$ rel. max, $g(-3)=-4$ rel. mini of g.

Now $g''(x)=3(x+5)+(3x+9)=6x+24=0$

$\Rightarrow x=-4$

		-4	
g''	$-$		$+$
	C.D.		C.U.

$(-4,-2)$: inflection point. $g(-4)=-2$

(b) $f'(x)=-4(1+x^2)^{-2}(2x)=-8x(1+x^2)^{-2}$

$=0\Rightarrow x=0$: critical #

		0	
f'	$+$		$-$
f	$\nearrow$		$\searrow$

$f(0)=4$ is rel. max.

Now. $f''(x)=-8(1+x^2)^{-2}-8x(1+x^2)^{-3}(2x)$

$=\frac{8(3x^2-1)}{(1+x^2)^3}=0\Rightarrow x=\pm\frac{1}{\sqrt{3}}$

		$-\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{3}}$	
f''	$+$		$-$		$+$
f	C.U.		C.D.		C.U.

$f(\pm\frac{1}{\sqrt{3}})=3$

$(\frac{1}{\sqrt{3}},3)$, $(-\frac{1}{\sqrt{3}},3)$: inflection points

#6. $f(x) = x(x^2+1)^{-1}$, conti on $[0, 2]$

$\Rightarrow^{\text{set}} f'(x) = (x^2+1)^{-1} - x(x^2+1)^{-2}(2x)$

$= (x^2+1)^{-2}(x^2+1-2x^2) = \frac{1-x^2}{(x^2+1)^2} = 0$

$\Rightarrow x = -1, 1.$: critical points

$\Rightarrow x = 1$ the only critical point in $[0,2]$

Note $f(0) = 0$, $f(2) = \frac{2}{5}$ and
$\leftarrow$ smallest
$f(1) = \frac{1}{2}$
$\leftarrow$ largest.

Hence f has absolute max. at $(1, \frac{1}{2})$ and abs. min. at $(0,0)$.

#5.

$f'(x) = -x^2 + 2x - 1$

$= -(x^2-2x+1) = -(x-1)^2$

$= 0 \Rightarrow x = 1.$

$\Rightarrow$ f': $-$ 1 $-$; f: $\downarrow$ 1 $\downarrow$ — $f \downarrow$ on R $\Rightarrow$ No rel extrema

$f''(x) = -2(x-1) = 0 \Rightarrow x = 1$

f'': $+$ 1 $-$

$\Rightarrow$ f: C.U. 1 C.D.

f': $\uparrow$ $\downarrow$

$\Rightarrow$

(a) f' is $\uparrow$ on $(-\infty, 1)$ and $\downarrow$ on $(1, \infty)$

(b) f is C.U. on $(-\infty, 1)$ and C.D. on $(1, \infty)$, and

(c) hence f has a point of inflection at $x = 1$

Also, f is $\downarrow$ on R. hence no relative extrema.