榮譽第一　sol. to midterm 2018.11.21

國立東華大學 應用數學系 ______學年度第____學期　考試科目：Calculus B(I) □期中□期末考試試卷

學號________姓名 Yes ______系所__________班別______任課教師：Yes ________共____張

1. f is diff at $x=c$ i.e. $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}=f'(c)$ exists. $\lim_{x\to c}(x-c)=0$

$\Rightarrow \lim_{x\to c}(f(x)-f(c))=\lim_{x\to c}\left[\frac{f(x)-f(c)}{x-c}(x-c)\right]$

$=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}\cdot\lim_{x\to c}(x-c)=f'(c)\cdot 0=0$

i.e. $\lim_{x\to c}f(x)=f(c)$ i.e. f : conti at $x=c$. ※

2. $f(x)=-\frac{1}{3}x^3+x^2-x+5 \Rightarrow f'(x)=-x^2+2x-1$

$=-(x^2-2x+1)=-(x-1)^2=0 \Rightarrow x=1$

the critical point　f′ − | − ; f ↘ 1 ↘

i.e. (a) f is ↘ in R

$f''(x)=-2(x-1)=0 \Rightarrow x=1$

f″ + | − ; f C.U. 1 C.D.

i.e. (b) f : CU on $(-\infty,1)$, CD on $(1,\infty)$

hence (c) $(1,f(1))$ is inflection point

and by (a) f has no extrema.

3. (a) $\lim_{x\to 0}(e^{-2x}+5x)^{\frac{1}{x}}$ = "1^{∞}"

$=\exp\left(\lim_{x\to 0}\frac{1}{x}\ln(e^{-2x}+5x)\right)=\boxed{e^3}$

∵ $\lim_{x\to 0}\frac{\ln(e^{-2x}+5x)}{x}$ = "$\frac{0}{0}$"

$=\lim_{x\to 0}\frac{\frac{-2e^{-2x}+5}{e^{-2x}+5x}}{1}=\frac{-2+5}{1}=3$

(b) $\lim_{x\to 0}\frac{\sin(9x)}{\sin(3x)}$ = "$\frac{0}{0}$"

$=\lim_{x\to 0}\frac{9\cos(9x)}{3\cos(3x)}=\frac{9}{3}=3$

(c) $\lim_{t\to\infty}t^3e^{-6t}=\lim_{t\to\infty}\frac{t^3}{e^{6t}}$ = "$\frac{\infty}{\infty}$"

$=\lim_{t\to\infty}\frac{3t^2}{6e^{6t}}$

$=\lim_{t\to\infty}\frac{6t}{36e^{6t}}=\lim_{t\to\infty}\frac{6}{216e^{6t}}=0$

#4 $2x+xy-2=\ln(x^3+y^2)$

$\frac{d}{dx}\Rightarrow$ $2+y+x\frac{dy}{dx}=\frac{3x^2+2y\frac{dy}{dx}}{x^3+y^2}$

i.e. $2+y-\frac{3x^2}{x^3+y^2}=(2y-x)\frac{dy}{dx}$

$\frac{dy}{dx}=\frac{2+y-\frac{3x^2}{x^3+y^2}}{2y-x}\Big|_{x=1,\,y=0}$

$=\frac{2-3}{-1}=1$ = slope

∴ tangent line $y-0=1(x-1)$ i.e. $y=x-1$

#5 $h(t)=(e^t+e^{-t})^3$, $-1\le t\le 3$

$\Rightarrow h'(t)=3(e^t+e^{-t})^2\cdot(e^t-e^{-t})=0$

$\Rightarrow e^t-e^{-t}=0 \Rightarrow t=0$

h′ − | + ; ↘ 0 ↗ ⇒ h(0) = min.

$h(0)=2^3=8$　$h(-1)=(e+e^{-1})^3$

$h(3)=(e^3+e^{-3})^3>h(1)=(e^{-1}+e)^3=h(-1)$

$\Rightarrow h(0)=8$ abs. min. $h(3)=(e^3+e^{-3})^3$ abs. max

#6 (a) $e^{2x-x^5}\log_7 y=5\sin(2x)+y^2\ln((3x^2+1)^2)$

$e^{2x-x^5}\frac{\ln y}{\ln 7}$　　$5\sin(2x)+2y^2\ln(3x^2+1)$

$\frac{d}{dx}\Rightarrow$

$e^{2x-x^5}(2-5x^4)\frac{\ln y}{\ln 7}+e^{2x-x^5}\frac{1}{\ln 7}\frac{1}{y}\frac{dy}{dx}$

$=5\cos(2x)\times 2+4y\cdot\frac{dy}{dx}\ln(3x^2+1)+2y^2\frac{6x}{3x^2+1}$

$\Rightarrow \frac{dy}{dx}=\frac{10\cos(2x)+2y^2\frac{6x}{3x^2+1}-e^{2x-x^5}(2-5x)\log_7 y}{e^{2x-x^5}\frac{1}{\ln 7}\frac{1}{y}-4y\cdot\ln(3x^2+1)}$

(b). $f(x)=x^x3^{x^2} \Rightarrow \ln f(x)=x\ln x+x^2\ln 3$

$\frac{d}{dx}\ln f(x)=x\cdot\frac{1}{x}+\ln x\cdot 1+(2\ln 3)x$

$\frac{1}{f(x)}f'(x)$　　$1+\ln x+2x\ln 3$

$\Rightarrow f'(x)=x^x3^{x^2}\cdot(1+\ln x+2x\ln 3)$

#6

(c) $\because \tan'(x) = \sec^2(x)$

$\Rightarrow f(x) = 3\tan^2(\ln(\sqrt{5x}))$

$\Rightarrow f'(x) = 6\tan(\ln(\sqrt{5x}))\cdot\sec^2(\ln(\sqrt{5x}))\cdot\frac{1}{\sqrt{5x}}(5x)^{-\frac{1}{2}}5$

$= 6\tan(\ln\sqrt{5x})\sec^2(\ln(\sqrt{5x}))\frac{1}{x}$

(d) $\ln y = 6\ln(2x^3) - 12x + 5\ln(3x^2 - x + 2) + 2.3\ln(1 + \cos(4x^2) + x^8)$

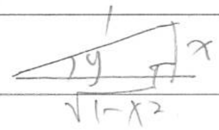
$\frac{d}{dx} \Rightarrow \frac{1}{y}\frac{dy}{dx} = 6\frac{6x^2}{2x^3} - 12 + 5\cdot\frac{6x-1}{3x^2-x+2} + 2.3\frac{(-\sin(4x^2))\cdot 8x + 8x^7}{1+\cos(4x^2)+x^8}$

$= \frac{18}{x} - 12 + \frac{30x-5}{3x^2-x+2} + 2.3\frac{8x(-\sin(4x^2)+x^6)}{1+\cos(4x^2)+x^8}$

$= A$

$\frac{dy}{dx} = y*A = A*\frac{(2x^3e^{-2x})^6(3x^2-x+2)^5}{[1+\cos(4x^2)+x^8]^{-2.3}}$

(e) $y = \sin^{-1}(x)$



$\sin y = x$

$\cos y = \sqrt{1-x^2}$

$\Leftrightarrow \sin(y) = x$

$\frac{d}{dx} \Rightarrow \cos(y)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos(y)}$

$= \frac{1}{\sqrt{1-x^2}}$

#7. $f(x) = \int f'(x)\,dx = \int 1 + e^x + \frac{1}{x}\,dx$

$= x + e^x + \ln|x| + C$

$\Rightarrow f(1) = 1 + e + C = e + 3 \Rightarrow C = 2$

$\Rightarrow f(x) = x + e^x + \ln|x| + 2$