榮譽第一

sol to Quiz #4 on 12-28

國立東華大學
應用數學系____學年度第___學期 考試科目：________ □期中□期末考試試卷

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#1. The average value of f on [1, 5]

$= \frac{1}{5-1}\int_1^5 f(x)\,dx = \frac{1}{4}\int_1^5 \frac{3}{x+2}\,dx$

$= \frac{3}{4}\ln(|x+2|)\Big|_1^5 = \frac{3}{4}(\ln(7)-\ln(3))$

#2. $\Rightarrow f(x) = \frac{3}{x+2} = \frac{3}{4}\ln\left(\frac{7}{3}\right) \Rightarrow x = \frac{4}{\ln 7 - \ln 3} - 2$

(a) $\int_0^1 e^{2x}\sqrt{e^{2x}+1}\,dx$ $\quad u = e^{2x}+1 \Rightarrow du = 2e^{2x}dx \Rightarrow e^{2x}dx = \frac{du}{2}$

$= \int_2^{e^2+1} \frac{1}{2}u^{\frac{1}{2}}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}\,u^{\frac{3}{2}}\Big|_2^{e^2+1} = \frac{1}{3}(e^2+1)^{\frac{3}{2}} - \frac{1}{3}2^{\frac{3}{2}}$

(b)

$\int_1^4 |x-2|\,dx = \int_1^2 2-x\,dx + \int_2^4 x-2\,dx$

$= 2x\Big|_1^2 - \frac{1}{2}x^2\Big|_1^2 + \frac{1}{2}x^2\Big|_2^4 - 2x\Big|_2^4$

$= 2 \cancel{-2} + \frac{1}{2} \cancel{+8} \cancel{-2} \cancel{-8} + 4$

$= 2.5$

(c)

$\int x^3\ln x\,dx = \int \frac{1}{4}\ln x\,dx^4$

$= \frac{1}{4}x^4\ln x - \int x^4\,\frac{1}{4}\,\frac{1}{x}\,dx$

$= \frac{1}{4}x^4\ln x - \frac{1}{4}\int x^3\,dx$

$= \frac{1}{4}x^4\ln x - \frac{1}{16}x^4 + C$

$\Rightarrow \int_1^e x^3\ln x\,dx = \frac{1}{4}x^4\ln x - \frac{1}{16}x^4\Big|_1^e$

$= \frac{1}{4}e^4 - \frac{1}{16}e^4 - \frac{1}{4}\cdot 1\cdot 0 + \frac{1}{16}\cdot 1$

$= \frac{3}{16}e^4 + \frac{1}{16}$

(d)

$\int_1^2 x^2e^x\,dx = \int_1^2 x^2\,de^x$

$= x^2e^x\Big|_1^2 - \int_1^2 2e^x x\,dx$

$= 4e^2 - e - \int_1^2 2x\,de^x$

$= 4e^2 - e - 2xe^x\Big|_1^2 + 2\int_1^2 e^x\,dx$

$= 4e^2 - e - 4e^2 + 2e + 2e^x\Big|_1^2$

$= e + 2e^2 - 2e = 2e^2 - e$

$= e(2e-1)$

(e) $\int x(\ln x)^2\,dx = \int (\ln x)^2\,d\frac{x^2}{2}$

$= \frac{1}{2}x^2(\ln x)^2 - \int \frac{x^2}{2}\cdot 2\ln x\cdot\frac{1}{x}\,dx$

$= \frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx$

$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2 + C$

since $\int x\ln x\,dx = \int \ln x\,d\frac{x^2}{2}$

$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\,\frac{1}{x}\,dx$

$= \frac{x^2}{2}\ln x - \int \frac{1}{2}x\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$

(f)

$\int \frac{xe^{2x}}{(2x+1)^2}\,dx = \int xe^{2x}(2x+1)^{-2}\,dx$

$= \int -\frac{xe^{2x}}{2}\,d(2x+1)^{-1}$

$= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{1}{2(2x+1)}(e^{2x} + 2xe^{2x})\,dx$

$= -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{2}\int e^{2x}\,dx$

$= -\frac{xe^{2x}}{2(2x+1)} + \frac{1}{4}e^{2x} + C$

$= \frac{e^{2x}}{4(2x+1)}(2x+1-2x) + C$

$= \frac{e^{2x}}{4(2x+1)} + C$

(g)

$\int \frac{x^2}{(3x-5)^2}\,dx$ $\quad u = 3x-5 \Rightarrow du = 3dx \Rightarrow dx = \frac{1}{3}du,\ x = \frac{u+5}{3}$

$= \int \frac{(u+5)^2}{9u^2}\,\frac{1}{3}\,du$

$= \frac{1}{27}\int \frac{u^2+10u+25}{u^2}\,du$

$= \frac{1}{27}\int 1 + 10\,\frac{1}{u} + 25u^{-2}\,du$

$= \frac{1}{27}\left(u + 10\ln(|u|) - 25u^{-1}\right) + C$

$= \frac{1}{27}\left(3x-5 + 10\ln(|3x-5|) - \frac{25}{3x-5}\right) + C$

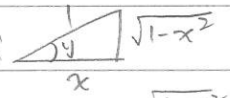
(h) $\displaystyle\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$ $\quad x=\cos y$

$\quad dx = -\sin y\,dy$

$= \displaystyle\int \frac{1}{\cos^2 y \cdot \sin y}(-\sin y)\,dy$

$= -\displaystyle\int \sec^2 y\,dy$

$= -\displaystyle\int d\tan y$ $\quad \because$



$= -\tan y + C$ $\quad \Rightarrow \tan y = \dfrac{\sqrt{1-x^2}}{x}$

$= -\dfrac{\sqrt{1-x^2}}{x} + C$

(i) $\displaystyle\int 6x\sec^2 x\,dx$

$= \displaystyle\int 6x\,d\tan x$

$= 6x\tan x - \displaystyle\int 6\tan x\,dx$

$= 6x\tan x - 6\displaystyle\int \frac{\sin x}{\cos x}\,dx$

$= 6x\tan x + 6\displaystyle\int \frac{1}{\cos x}\,d\cos x$

$= 6x\tan x + 6\ln(|\cos x|) + C$