榮譽第一　　Sol. to Cal. AJ. Quiz #3　　1214

國立東華大學
應用數學系 ______ 學年度第 ___ 學期　考試科目：________________ □期中□期末考試試卷

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#1. (a) Let $u = 1-x^2 \Rightarrow du = -2x\,dx$

$\Rightarrow x\,dx = -\frac{1}{2}du$

$\Rightarrow \int 5x\sqrt[3]{1-x^2}\,dx = \int 5x(1-x^2)^{\frac{1}{3}}dx$

$= -5/2\int u^{\frac{1}{3}}\,du = -\frac{5}{2}\,\frac{1}{\frac{1}{3}+1}\,u^{\frac{1}{3}+1}+C$

$= -\frac{15}{8}(1-x^2)^{\frac{4}{3}}+C$

(b) $u = x^3+3x+4 \Rightarrow du = (3x^2+3)dx$

$\Rightarrow (x^2+1)dx = \frac{1}{3}du$

$\Rightarrow \int \frac{x^2+1}{\sqrt{x^3+3x+4}}dx = \frac{1}{3}\int u^{-\frac{1}{2}}du$

$= \frac{1}{3}\,\frac{1}{-\frac{1}{2}+1}\,u^{-\frac{1}{2}+1}+C = \frac{2}{3}\sqrt{x^3+3x+4}+C$

(c) $u = \ln x \Rightarrow du = \frac{1}{x}dx$

$\Rightarrow \int \frac{\ln x}{x}dx = \int u\,du = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$

(d) $u = 1-e^{-x} \Rightarrow du = e^{-x}dx$

$\Rightarrow \int \frac{e^{-x}}{1-e^{-x}}dx = \int \frac{1}{u}du$

$= \ln(|u|)+C = \ln(|1-e^{-x}|)+C$

(e) $\int \frac{e^{2x}+2e^{x}+1}{e^{x}}dx$

$= \int e^{x}dx + 2\int dx + \int e^{-x}dx$

$= e^{x}+2x-\int de^{-x}$

$= e^{x}+2x-e^{-x}+C$

(f) $\int \frac{1+e^{-x}}{1+xe^{-x}}dx = \int \frac{e^{x}}{e^{x}}\,\frac{1+e^{-x}}{1+xe^{-x}}dx$

$= \int \frac{e^{x}+1}{e^{x}+x}dx$　　$u = e^{x}+x$

$= \int \frac{1}{u}du$　　$\Rightarrow du = (e^{x}+1)dx$

$= \ln(|u|)+c = \ln(|e^{x}+x|)+c$

#2.

$f(x) = \int f'(x)\,dx = \int \frac{x^2+4x+3}{x-1}dx$

$= \int x+5+\frac{8}{x-1}dx$

$= \frac{1}{2}x^2+5x+8\int \frac{1}{x-1}dx$

$= \frac{1}{2}x^2+5x+8\int \frac{1}{x-1}d(x-1)$

$= \frac{1}{2}x^2+5x+8\ln(|x-1|)+c$

∵
$$\begin{array}{r} x+5 \\ x-1\,\overline{)\,x^2+4x+3} \\ \underline{x^2-x} \\ 5x+3 \\ \underline{5x-5} \\ 8 \end{array}$$

But $f(2) = \frac{1}{2}\cdot 4+10+8\ln(1)+C = 12+C$

$= 4 \Rightarrow C = -8$

$\therefore f(x) = \frac{1}{2}x^2+5x+8\ln(|x-1|)-8.$

#3. $f'(x) = \int f''(x)\,dx = \int x^{-\frac{2}{3}}dx$

$= 3x^{\frac{1}{3}}+C$

w/ $f'(8) = 3\cdot 8^{\frac{1}{3}}+C = 6+C = 6 \Rightarrow C=0$

$\Rightarrow f'(x) = 3x^{\frac{1}{3}}$

$\Rightarrow f(x) = \int f'(x)\,dx = \int 3x^{\frac{1}{3}}dx$

$= 3\cdot\frac{3}{4}x^{\frac{4}{3}}+C = \frac{9}{4}x^{\frac{4}{3}}+C$

w/ $f(0) = C = 0 \Rightarrow C = 0$

$\therefore f(x) = \frac{9}{4}x^{\frac{4}{3}}$

#4. (a) $u = \sec x - 1 \Rightarrow du = (\sec x\cdot\tan x)dx$

$\Rightarrow \int \frac{\sec x\cdot\tan x}{\sec x-1}dx = \int \frac{1}{u}du$

$= \ln(|u|)+C = \ln(|\sec x-1|)+C$

(b) $\int (\sin x+\cos x)^2dx$

$= \int \sin^2x+2\sin x\cos x+\cos^2x\,dx$

$= \int 1+2\sin x\cos x\,dx$　　$\because \sin^2x+\cos^2x = 1$

$= \int dx+2\int \sin x\cdot d\sin x$　　$\because d\sin x = \cos x\,dx$

$= x+\sin^2x+C$