榮譽第一

國立東華大學 應用數學系 ______學年度第____學期　考試科目：Cal AI Quiz#2 1102.　□期中□期末考試試卷

學號______ 姓名 yes 系所______ 班別____ 任課教師：yes 共 1/2 張

#1. $f(x) = (x^2+8)(x-1)^{-1}$

$\overset{set}{\Rightarrow} f'(x) = 2x(x-1)^{-1} - (x-1)^{-2}(x^2+8)$

$= (x-1)^{-2}(2x(x-1) - x^2 - 8)$

$= \dfrac{x^2-2x-8}{(x-1)^2} = \dfrac{(x-4)(x+2)}{(x-1)^2}$

$= 0 \overset{At}{\Rightarrow} x = 4$ or $x = -2$

$f(x)$ has horizontal tangent line.

note $f(4) = 8$, $f(-2) = -4$.

Hence f has horizontal tangent at points $(-2,-4)$ and $(4,8)$.

#2

(a) $f'(x) = \dfrac{d}{dx}\left[(3x^3+4x)^{\frac{1}{5}}\right]$

$= \frac{1}{5}(3x^3+4x)^{-\frac{4}{5}}(9x^2+4)$

$\Rightarrow f'(2) = \frac{1}{5}(3\times 2^3 + 4\times 2)^{-\frac{4}{5}} \times (9\times 2^2+4)$

$= \frac{1}{5} 2^{-4} \times 40 = \frac{1}{5}\times\frac{1}{16}\times 40 = \frac{1}{2}$

$\therefore$ Tangent line at $(2,2)$ is

$y-2 = f'(2)(x-2)$ i.e. $y = \frac{1}{2}x + 1$

(b) $f'(x) = \dfrac{d}{dx}\left[(x+1)(2x-3)^{-\frac{1}{2}}\right]$

$= (2x-3)^{-\frac{1}{2}} - \frac{1}{2}(2x-3)^{-\frac{3}{2}}(2)(x+1)$

$= (2x-3)^{-\frac{3}{2}}\left((2x-3)-(x+1)\right)$

$= (2x-3)^{-\frac{3}{2}}(x-4) \Rightarrow f'(2) = -2$

$\therefore$ Tangent line at $(2,3)$ is

$y-3 = -2(x-2)$ i.e. $y = -2x+7$

(c) $\dfrac{d}{dx}[\sin(x) + \cos(2y)] = \dfrac{d}{dx}(1)$

$\Leftrightarrow \cos(x) - \sin(2y)\, 2\dfrac{dy}{dx} = 0$

$\therefore \dfrac{dy}{dx} = \dfrac{\cos(x)}{2\sin(2y)} = 0$ at $x = \frac{\pi}{2}$

hence, tangent line at $(\frac{\pi}{2}, \frac{\pi}{4})$ is

$y - \frac{\pi}{4} = 0\cdot(x - \frac{\pi}{2})$ i.e. $y = \frac{\pi}{4}$

#3. (a) $f(u) = (3u-1)^{-2} \Rightarrow f'(u) = -2(3u-1)^{-3}\times 3$

$\Rightarrow f''(u) = -6\times(-3)(3u-1)^{-4}(3)$

$= 54(3u-1)^{-4}$

$\Rightarrow f'''(u) = 54\times(-4)(3u-1)^{-5}\cdot(3)$

$= \dfrac{-648}{(3u-1)^5}$

(b) $f'(x) = 3(x^3+x^2)^2\cdot(3x^2+2x)$

$\Rightarrow f''(x) = 3\times 2(x^3+x^2)(3x^2+2x)^2$

$+ 3\times(x^3+x^2)^2(6x+2)$

$\therefore f''(1) = 6\times 2\times 5^2 + 3\times 2^2\times 8 = 396$

#4. (a) $g(x) = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$, $x > 0$

$\Rightarrow g'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{3}{2}} = x^{-\frac{3}{2}}(\frac{x}{2} - 2)$

$= 0 \Rightarrow x = 4$: only critical point.

	0		4	
g'		−		+
g		↘		↗

$\overset{At}{\Rightarrow} x = 4$ g has rel. min $g(4) = \sqrt{4} + 4\cdot\frac{1}{\sqrt{4}} = 4$

Now, $g''(x) = -\frac{1}{4}x^{-\frac{3}{2}} + 3x^{-\frac{5}{2}}$

$= x^{-\frac{5}{2}}(3 - \frac{1}{4}x) = 0$

$\Rightarrow x = 12$, note

	0		12	
g''		+		−
g		C.U. ∪		C.D. ∩

hence $(12, g(12)) = (12, 8\sqrt{3}/3)$ is point of inflection.

Note $\lim\limits_{x\to\infty} g(x) = \infty$, $\lim\limits_{x\to 0^+} g(x) = \infty$

hence g has no rel. max.

(b) $g(x) = x(x+3)^{\frac{1}{2}}$, $x \geq -3$

$\overset{set}{\Rightarrow} g'(x) = (x+3)^{\frac{1}{2}} + x\cdot\frac{1}{2}(x+3)^{-\frac{1}{2}}$

$= (x+3)^{-\frac{1}{2}}(x+3+\frac{x}{2})$

$= \frac{1}{2}(x+3)^{-\frac{1}{2}}(3x+6) = 0 \Rightarrow x = -2$

$g'(x)$ D.N.E. at $x = -3$

$\Rightarrow x = -3, -2$ are critical points.

	−3		−2	
g'		−		+
g		↘		↗

$\Rightarrow (-2, g(-2)) = (-2, -2)$ rel. min.

$(-3, g(-3)) = (-3, 0)$ rel. max.

(note.
(but $\lim_{x\to\infty} g(x) = \infty$ hence $(-3, 0)$
is not absolute max)

Now, $g'(x) = \frac{1}{2}(x+3)^{-\frac{1}{2}}(3x+6)$

$\Rightarrow g''(x) = -\frac{1}{4}(x+3)^{-\frac{3}{2}}(3x+6) + \frac{1}{2}(x+3)^{-\frac{1}{2}}(3)$

$= -\frac{1}{4}(x+3)^{-\frac{3}{2}}\left[3x+6 - 4\frac{3}{2}(x+3)\right]$

$= -\frac{1}{4}(x+3)^{-\frac{3}{2}}(-3x-12)$

$= \frac{3}{4}(x+3)^{-\frac{3}{2}}(x+4) = 0$

$\Rightarrow x = -4 \notin$ Domain of g

note $g''(x) > 0$, $\forall x > -3$

hence g is c.u $\forall x > -3$.

$\Rightarrow$ No point of inflection.

#5.

$f'(x) = -x^2 + 2x - 1$

$= -(x^2 - 2x + 1) = -(x-1)^2$

$= 0 \Rightarrow x = 1$

$\Rightarrow$ $\because f'$: $-$ | $-$ (at 1); $f \downarrow$ on $\mathbb{R}$.
$\therefore f$: $\downarrow$ 1 $\downarrow$ $\Rightarrow$ No rel extrema

$f''(x) = -2(x-1) = 0 \Rightarrow x = 1$

$\Rightarrow$ $\because f''$: $+$ | $-$ (at 1)
$\therefore f$: C.U. 1 C.D.
f': $\uparrow$ $\downarrow$

$\Rightarrow$
(a) f' is $\uparrow$ on $(-\infty, 1)$ and $\downarrow$ on $(1, \infty)$

(b) f is C.U. on $(-\infty, 1)$ and C.D. on $(1, \infty)$, and

(c) hence f has a point of inflection at $x = 1$

Also, f is $\downarrow$ on $\mathbb{R}$. hence no relative extrema.

#6 $f(x) = x(x^2+1)^{-1}$, conti on $[0, 2]$

$\overset{sol}{\Rightarrow} f'(x) = (x^2+1)^{-1} - x(x^2+1)^{-2}(2x)$

$= (x^2+1)^{-2}(x^2+1-2x^2) = \frac{1-x^2}{(x^2+1)^2} = 0$

$\Rightarrow x = -1, 1$: critical points

$\Rightarrow x = 1$ the only critical point in $[0,2]$

Note $f(0) = 0$ ← smallest, $f(2) = \frac{2}{5}$ and
$f(1) = \frac{1}{2}$ ← largest

Hence f has absolute max. at $(1, \frac{1}{2})$ and abs. min. at $(0, 0)$.