Sol. to. Midterm. 11/23

榮譽第一

國立東華大學 應用數學系 ______學年度第___學期 考試科目：______________ □期中□期末考試試卷

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#1. see sol. to Quiz 1

#2. see sol to Quiz 2

#3.

(a) $\lim_{x\to 1} \frac{\ln(x)}{x^2-1}$ is of " $\frac{0}{0}$ " form

$= \lim_{x\to 1} \frac{\frac{1}{x}}{2x} = \frac{1}{2}$.

(b) $\lim_{x\to\infty} x^5 e^{-3x} = \lim_{x\to\infty} \frac{x^5}{e^{3x}}$ of " $\frac{\infty}{\infty}$ " form

$= \lim_{x\to\infty} \frac{5x^4}{3e^{3x}} = \lim_{x\to\infty} \frac{5\times4\times x^3}{3^2 e^{3x}}$ " $\frac{\infty}{\infty}$ "

$= \lim_{x\to\infty} \frac{5\times4\times3\times2\times1}{3^5 e^{3x}} = 0$

#4. Find $\frac{dy}{dx}$, $\because x+y-1 = \ln(x^2+y^2)$

$\therefore \frac{d}{dx}(x+y-1) = \frac{d}{dx}\ln(x^2+y^2)$

$1 + \frac{dy}{dx} \overset{\shortparallel}{} \frac{1}{x^2+y^2}(2x+2y\frac{dy}{dx})$

$\Rightarrow (1-\frac{2y}{x^2+y^2})\frac{dy}{dx} = \frac{2x}{x^2+y^2} - 1$

$\Rightarrow \frac{dy}{dx} = (\frac{2x}{x^2+y^2}-1)/(1-\frac{2y}{x^2+y^2})$

at $(1,0)$ $= (\frac{2}{1}-1)/(1-\frac{0}{1+0}) = 1$

= slope of the tangent line to $x+y-1 = \ln(x^2+y^2)$ at the point $(1,0)$

$\Rightarrow$ Its eq. is $y-0 = 1.(x-1)$

i.e. $y = x-1$

#5. $f(t) = 3t^5 - 5t^3$ a polynomial hence is conti. on $[-2,0]$

$\Rightarrow$ Abs. extrema exist.

Sol $f'(t) = 15t^4 - 15t^2$

$= 15t^2(t^2-1) = 0$

$\Rightarrow t = 0, -1, 1$ but $1 \notin [-2,0]$

$\Rightarrow f(0) = 0$ $\underbrace{f(-1) = 2}_{\text{largest}}$, $\underbrace{f(-2) = -56}_{\text{smallest}}$

$\Rightarrow f(-2) = -56$ = abs min

$f(-1) = 2$ = abs. max of f on $[-2,0]$

#6. (a) Take $\frac{d}{dx}$ on the eq.

$\Rightarrow e^{x-x^3}\cdot\frac{dy}{dx} + y\cdot e^{x-x^3}(1-3x^2)$

$= 3 + 8y\ln(x^2+1)\frac{dy}{dx} + 4y^2\cdot\frac{2x}{x^2+1}$

$\Rightarrow \frac{dy}{dx}(e^{x-x^3} - 8y\ln(x^2+1))$

$= 3 + 8y^2\frac{x}{x^2+1} - ye^{x-x^3}(1-3x^2)$

$\Rightarrow \frac{dy}{dx} = \frac{3+8y^2\frac{x}{x^2+1} - ye^{x-x^3}(1-3x^2)}{e^{x-x^3} - 8y\ln(x^2+1)}$

(b) $f(x) = x^x\, 5^{x^2}$

$\ln f(x) = \ln x^x + \ln 5^{x^2}$

$= x\ln x + x^2\ln 5$

$\Rightarrow \frac{d}{dx}\ln f(x) = \frac{1}{f(x)}f'(x)$

$= \frac{d}{dx}[x\ln x + x^2\ln 5]$

$= \ln x + x\cdot\frac{1}{x} + 2x\cdot\ln 5$

$= \ln x + 1 + 2x\cdot\ln 5$

$\Rightarrow f'(x) = f(x)\cdot[\ln x + 1 + 2x\ln 5]$

$= x^x 5^{x^2}(\ln x + 1 + 2x\ln 5)$

(c) $\ln y = \ln(3x^2+e^{4x})^3 + \ln e^{-4x}$

$- \ln(1+\cos(x^3)+x^2)^{\frac{2}{3}}$

$= 3\ln(3x^2+e^{4x}) - 4x$

$-\frac{2}{3}\ln(1+\cos(x^3)+x^2)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left[3\,\frac{6x+4e^{4x}}{3x^2+e^{4x}} - 4 - \frac{2}{3}\,\frac{-\sin(x^3)\cdot 3x^2 + 2x}{1+\cos(x^3)+x^2}\right]$

$\triangleq A \Rightarrow \frac{dy}{dx} = y\cdot A$

(d) $f(x) = \frac{e^{-x^2}+x}{\log_{10}x} = \frac{(e^{-x^2}+x)\ln 10}{\ln x}$

$\Rightarrow f'(x) = \frac{\ln x\cdot\ln 10(e^{-x^2}(-2x)+1) - (e^{-x^2}+x)(\ln 10)\frac{1}{x}}{(\ln x)^2}$

$= \frac{1-2xe^{-x^2}}{\log_{10}x} - \frac{(e^{-x^2}+x)\cdot(\ln 10)}{x(\ln x)^2}$

#7. Suppose f is diff. at $x=a$

i.e. $f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

exists

$\Rightarrow \lim_{x \to a} [f(x) - f(a)]$

$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a} \cdot (x-a)$

$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a} \cdot \lim_{x \to a} (x-a)$

$\because$ both limits exist.

$= f'(a) \cdot 0 = 0$

hence $\lim_{x \to a} f(x) = f(a)$

i.e. f is conti. at $x=a$

Thus, a differentiable ft. is continuous