

- problem 13.1.1 of the textbook.
- Let  $X$  be a discrete random variable whose p.m.f. is for  $k = 1, 2, \dots, 7$ ,  $P_p(X = k) = p_k$  where  $p = (p_1, p_2, \dots, p_7)$  with  $p_k \geq 0$ ,  $\forall k$  and  $\sum p_k = 1$ . Based on observing one  $X$ , give THREE different level  $\alpha = 0.05$  tests for testing  
 $H_0 : p = (0.01, 0.01, 0.02, 0.01, 0.01, 0.01, 0.93)$  v.s.  
 $H_1 : p = (0.06, 0.05, 0.04, 0.03, 0.02, 0.01, 0.79)$ .  
 Compute the probability of Type-II error for your tests.  
 Do the same with level 0.035.

- Let  $X \sim \text{Gamma}(1, \beta)$ . To test  $H_0 : \beta = 1$  v.s.  $H_1 : \beta > 1$ , suppose student A uses the non-randomized test

$$\varphi_A(x) = \begin{cases} 1 & \text{if } x > c, \\ 0 & \text{if } x \leq c; \end{cases}$$

and B uses

$$\varphi_B(x) = \begin{cases} 1 & \text{if } x < k, \\ 0 & \text{if } x \geq k; \end{cases}$$

Find the constants  $c$  and  $k$  so that both tests have the same size 0.05 and then derive and compare their power functions. Which test is better? Why?

- Let  $X_1, \dots, X_n$  be *i.i.d.*  $U(0, \theta)$  r.v.'s and  $X_{(n)} = \max\{X_1, \dots, X_n\}$ . Also let  $\theta_0, \theta_1$  with  $0 < \theta_0 < \theta_1$  and  $0 < \alpha < 1$  be given constants. For testing  $H_0 : \theta = \theta_0$  v.s.  $H_1 : \theta = \theta_1$ , Consider the test defined by

$$\varphi_1(x_1, \dots, x_n) = \varphi_1(x_{(n)}) = \begin{cases} 1 & \text{if } \theta_0 < x_{(n)}, \\ \alpha & \text{otherwise.} \end{cases}$$

Show that it is a level  $\alpha$  test and calculate its power.

Now, define another test  $\varphi_2$  as

$$\varphi_2(x_1, \dots, x_n) = \varphi_2(x_{(n)}) = \begin{cases} 1 & \text{if } k \leq x_{(n)}, \\ 0 & \text{otherwise,} \end{cases}$$

where  $k$  is determined such that  $E_{\theta_0}[\varphi_2(X_{(n)})] = \alpha$ .

Determine the value of  $k$ , and show that  $\varphi_2$  has the same power as  $\varphi_1$