1. Suppose that $X_1, \ldots, X_n$ are *i.i.d.* r.v.'s from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Given μ_0 and $0 < \alpha < 1$. Derive the level α LRT (likelihood ratio test) for testing $H_0 : \mu \leq \mu_0 \quad v.s. \quad H_1 : \mu > \mu_0$.

2. Suppose that $X_1, \ldots, X_n$ are *i.i.d.* r.v.'s from $N(\mu_x, 1)$, and $Y_1, \ldots, Y_m$ *i.i.d.* r.v.'s from $N(\mu_y, 1)$. Also assume that the X's are independent of the Y's. Derive the level α $(0 < \alpha < 1)$ LRT for testing $H_0 : \mu_x = \mu_y \quad v.s. \quad H_1 : \mu_x \neq \mu_y$.

3. Suppose that $X_1, \ldots, X_n$ are *i.i.d.* r.v.'s from $N(\mu_1, \sigma_1^2)$, and $Y_1, \ldots, Y_m$ *i.i.d.* r.v.'s from $N(\mu_2, \sigma_2^2)$. Also assume that the X's are independent of the Y's, and $\mu_1, \sigma_1^2, \mu_2, \sigma_2^2$ are all unknown.
Derive the level α $(0 < \alpha < 1)$ LRT for testing $H_0 : \sigma_1^2 = \sigma_2^2 \quad v.s. \quad H_1 : \sigma_1^2 \neq \sigma_2^2$.

4. Let $f(x; \theta) = \theta e^{-\theta x}$, for $x > 0$, and $f(x; \theta) = 0$, otherwise. Suppose that $X_1, \ldots, X_n$ are *i.i.d.* r.v.'s from the common probability density function $f(x; \theta_1)$, and $Y_1, \ldots, Y_m$ *i.i.d.* r.v.'s from $f(x; \theta_2)$. Also assume that the X's are independent of the Y's.
For testing $H_0 : \theta_1 = \theta_2 \quad v.s. \quad H_1 : \theta_1 \neq \theta_2$, given $0 < \alpha < 1$,
   (a) show that the LRT can be based on the statistic
   $$T = \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n X_i + \sum_{j=1}^m Y_j} ,$$
   (b) find the distribution of T when H_0 is true, then
   (c) show how the level α LRT can become an F test.

5. (FYI) A random sample $X_1, \ldots, X_n$ is drawn from a Pareto population with p.d.f.
$$f(x; \theta, \mu) = \frac{\theta \mu^\theta}{x^{\theta+1}} I_{[\mu, \infty)}(x),$$
where both $\theta > 0$ and $\mu > 0$ are unknown parameters. Show that a LRT for testing $H_0 : \theta = 1 \quad v.s. \quad H_1 : \theta \neq 1$ rejects H_0 if $T(\mathbf{x}) \leq c_1$ or $T(\mathbf{x}) \geq c_2$, where $0 < c_1 < c_2$,
$$T(\mathbf{X}) = \log \left[\frac{\prod_{i=1}^n X_i}{X_{(1)}^n} \right] ,$$
and $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$.

6. (FYI) Suppose that $X_1, \ldots, X_n$ are *i.i.d.* r.v.'s with a $Beta(\mu, 1)$ p.d.f. and $Y_1, \ldots, Y_m$ *i.i.d.* r.v.'s with a $Beta(\lambda, 1)$ p.d.f.. Also assume that the X's are independent of the Y's.
For testing $H_0 : \mu = \lambda \quad v.s. \quad H_1 : \mu \neq \lambda$, given $0 < \alpha < 1$,
   (a) show that the LRT (likelihood ratio test) can be based on the statistic
   $$T = \frac{\sum_{i=1}^n \log X_i}{\sum_{i=1}^n \log X_i + \sum_{j=1}^m \log Y_j} ,$$
   (b) find the distribution of T when H_0 is true, then show how to get a level $\alpha = 0.1$ LRT.