1. A sample X of size 1 is taken from the *p.d.f.*

$$f(x;\theta) = \begin{cases} \frac{2}{\theta^2}(\theta - x) & \text{if } 0 < x < \theta, \\ 0 & \text{otherwise.} \end{cases}$$

   Let θ_0, θ_1 with $\theta_0 > \theta_1$ be two given constants.

   (a) Find the most powerful (MP) level $\alpha = 0.05$ test for testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$. Calculate its power.

   (b) Write out the test in part (a) for the case where $\theta_0 = 2$ and $\theta_1 = 1$.

2. Let X be random variable with *p.d.f.* f whoch can be either f_0 or else f_1, where f_0 is the *p.d.f.* of $N(0,1)$ and f_1 is the *p.d.f.* of Cauchy(0, 1) (i.e. $f_1(x) = 1/(\pi(1+x^2))$, $x \in R$). Find the MP level α test for testing $H_0 : f = f_0$ v.s. $H_1 : f = f_1$. Calculate its power.

3. Let $X \sim f(x) = e^{-x}$, $x > 0$. Based on one observation of $Y = \theta X$, derive the MP level $\alpha = 0.1$ test for testing $H_0 : \theta = 1$ v.s. $H_1 : \theta = 2$. Compute its probability of Type-II error.

4. Let $X_1, \ldots, X_n$ be *i.i.d.* $U(0,\theta)$ r.v.'s and $X_{(n)} = \max\{X_1, \ldots, X_n\}$. Also let θ_0, θ_1 with $0 < \theta_0 < \theta_1$ and $0 < \alpha < 1$ be given constants . For testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$,

   (a) show that the test

$$\phi_1(x_1, \ldots, x_n) = \phi(x_{(n)}) = \begin{cases} 1 & \text{if } \theta_0 < x_{(n)}, \\ \alpha & \text{otherwise} \end{cases}$$

   is a MP level α test.

   (b) Define a test ϕ_2 as

$$\phi_2(x_1, \ldots, x_n) = \phi_2(x_{(n)}) = \begin{cases} 1 & \text{if } k \le x_{(n)}, \\ 0 & \text{otherwise,} \end{cases}$$

   where k is determined such that $E_{\theta_0}\phi_2(x_{(n)}) = \alpha$.
   Determine the value of k, and show that ϕ_2 is also a MP level α test for testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$.