1. Let $X \sim f(x) = e^{-x},\ x > 0$. Based on one observation of $Y = \theta X$, derive the MP level $\alpha = 0.1$ test for testing $H_0 : \theta = 1$ v.s. $H_1 : \theta = 2$. Compute its probability of Type-II error.

2. (Important example where MP test is not unique)
   Let $X_1, \ldots, X_n$ be *i.i.d.* $U(0, \theta)$ r.v.'s and $X_{(n)} = \max\{X_1, \ldots, X_n\}$. Also let θ_0, θ_1 with $0 < \theta_0 < \theta_1$ and $0 < \alpha < 1$ be given constants .
   For testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$,

   (a) show that the randomized test

   $$\phi_1(x_1, \ldots, x_n) = \phi_1(x_{(n)}) = \begin{cases} 1 & \text{if } x_{(n)} > \theta_0, \\ \alpha & \text{otherwise} \end{cases}$$

   is a MP level α test, calculate its power.

   (b) Define a non-randomized test ϕ_2 as

   $$\phi_2(x_1, \ldots, x_n) = \phi_2(x_{(n)}) = \begin{cases} 1 & \text{if } x_{(n)} \geq k, \\ 0 & \text{otherwise,} \end{cases}$$

   where k is determined such that $E_{\theta_0}\phi_2(x_{(n)}) = \alpha$.
   Determine the value of k, and show that ϕ_2 is also a MP level α test for testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$, by indicating it power is the same as the power $\phi_1(x_{(n)})$.

   (c) Define another non-randomized test ϕ_3 as

   $$\phi_3(x_1, \ldots, x_n) = \phi_3(x_{(n)}) = \begin{cases} 1 & \text{if } x_{(n)} > \theta_0 \text{ or } x_{(n)} \leq k, \\ 0 & \text{otherwise,} \end{cases}$$

   where k is determined such that $E_{\theta_0}\phi_3(x_{(n)}) = \alpha$.
   Determine the value of k, and show that ϕ_3 is also a MP level α test for testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$.

3. Let $\alpha \in (0, 1)$, $0 < \theta_0 < \theta_1$ be given and $X_1, \ldots, X_n$ be a random sample from $f(x; \theta)$ where

   $$f(x; \theta) = \theta/x^2,\ \theta \leq x < \infty.$$

   Derive the MP (most powerful) level α test for testing $H_0 :\ \theta = \theta_0\ v.s.\ H_1 :\ \theta = \theta_1$. Calculate its power.

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