

1. Let X_1, \dots, X_n be a random sample from $N(\mu, \sigma^2)$.
 - (a) If σ^2 is known, find a minimum sample size n to guarantee that the 95% two-sided UMAU confidence interval for μ will have length no more than $\sigma/4$.
 - (b) If σ^2 is unknown, how to find a minimum sample size n to guarantee, with probability 0.9, that the 95% two-sided UMAU confidence interval for μ will have length no more than $\sigma/4$?
2. Let a random variable $X \sim f(x; \theta)$, where f is a p.d.f. defined as

$$f(x; \theta) = \frac{e^{(x-\theta)}}{(1 + e^{(x-\theta)})^2}, \quad x \in R, \theta \in R.$$

Based on one observation, X , find the UMA one-sided $1 - \alpha$ confidence interval of the form $\{\theta : \theta \leq U(X)\}$.

3. Let X be a single observation from $\text{Beta}(\theta, 1)$, $\theta > 0$.
 - (a) Let $Y = -(\ln X)^{-1}$. Evaluate the confidence coefficient of the interval $[Y/2, Y]$, that is: calculate $\inf_{\theta > 0} P_{\theta}(\theta \in [Y/2, Y])$.
 - (b) Find a pivot-based confidence interval having the same confidence coefficient as the interval in part (a).