Instructor: Yu-Ling Tseng (Due on 20220331, in class.)

1. A sample of size 1 is taken from the p.d.f.

$$f(x; \theta) = \begin{cases} \frac{2}{\theta^2} (\theta - x) & \text{if } 0 < x < \theta, \\ 0 & \text{otherwise.} \end{cases}$$

Let  $\theta_0$ ,  $\theta_1$  with  $\theta_0 > \theta_1$  be two given constants.

- (a) Find the most powerful (MP) level  $\alpha = 0.05$  test for testing  $H_0: \theta = \theta_0$  v.s.  $H_1: \theta = \theta_1$ . Calculate its power.
- (b) Write out the test in part (a) for the case where  $\theta_0 = 2$  and  $\theta_1 = 1$ .
- 2. Let X be random variable with p.d.f. f whoch can be either  $f_0$  or else  $f_1$ , where  $f_0$  is the p.d.f. of N(0,1) and  $f_1$  is the p.d.f. of Cauchy(0, 1) (i.e.  $f_1(x) = 1/(\pi(1+x^2))$ ,  $x \in R$ ). Find the MP level  $\alpha$  test for testing  $H_0: f = f_0$  v.s.  $H_1: f = f_1$ . Calculate its power.
- 3. Let  $X \sim f(x) = e^{-x}$ , x > 0. Based on one observation of  $Y = \theta X$ , derive the MP level  $\alpha = 0.1$  test for testing  $H_0: \theta = 1$  v.s.  $H_1: \theta = 2$ . Compute its probability of Type-II error.
- 4. Let  $X_1, \ldots, X_n$  be i.i.d.  $U(0, \theta)$  r.v.'s and  $X_{(n)} = \max\{X_1, \ldots, X_n\}$ . Also let  $\theta_0, \theta_1$  with  $0 < \theta_0 < \theta_1$  and  $0 < \alpha < 1$  be given constants. For testing  $H_0: \theta = \theta_0$  v.s.  $H_1: \theta = \theta_1$ ,
  - (a) show that the test

$$\phi_1(x_1, \dots, x_n) = \phi(x_{(n)}) = \begin{cases} 1 & \text{if } \theta_0 < x_{(n)}, \\ \alpha & \text{otherwise} \end{cases}$$

is a MP level  $\alpha$  test.

(b) Define a test  $\phi_2$  as

$$\phi_2(x_1, \dots, x_n) = \phi_2(x_{(n)}) = \begin{cases} 1 & \text{if } k \le x_{(n)}, \\ 0 & \text{otherwise,} \end{cases}$$

where k is determined such that  $E_{\theta_0}\phi_2(x_{(n)}) = \alpha$ .

Determine the value of k, and show that  $\phi_2$  is also a MP level  $\alpha$  test for testing  $H_0: \theta = \theta_0$  v.s.  $H_1: \theta = \theta_1$ .