商用微積分小考　　　　　　　　　　　　　　　　　　　　教師：曾玉玲 (Yes)
日期：　　姓名：Yes.　　學號：　　　　　系別：

1. (60 points) Find

(a) $\displaystyle\int \frac{x^2+3}{(x^3+9x-4)^2}\,dx$　(b) $\displaystyle\int \frac{x^2+2x+5}{x-1}\,dx$　(c) $\displaystyle\int \frac{1}{x\ln x}\,dx$

(d) $\displaystyle\int \frac{1+e^{-x}}{1+xe^{-x}}\,dx$　(e) $\displaystyle\int \frac{e^{2x}+2e^{x}+1}{e^x}\,dx$　(f) $\displaystyle\int x(\ln x)^2\,dx$

(a) $u=x^3+9x-4$.
$du=(3x^2+9)dx$
$=3(x^2+3)dx$
$\Rightarrow \displaystyle\int \frac{x^2+3}{(x^3+9x-4)^2}dx$
$=\frac{1}{3}\int u^{-2}du$
$=\frac{1}{3}\cdot\frac{1}{-2+1}u^{-2+1}+c$
$=-\frac{1}{3}\frac{1}{x^3+9x-4}+c$

(b) $=\displaystyle\int \frac{x^2-x+3x-3+8}{x-1}dx$
$=\int x\,dx+3\int dx+\int\frac{8}{x-1}dx$
$=\frac{1}{2}x^2+3x+8\ln|x-1|+c$

(c) $u=\ln x$
$du=\frac{1}{x}dx$
$\displaystyle\int\frac{1}{x\ln x}dx$
$=\int\frac{1}{u}du$
$=\ln|u|+c$
$=\ln|\ln x|+c$

(d) $=\displaystyle\int\frac{e^x\cdot(1+e^{-x})}{e^x(1+xe^{-x})}dx$
$=\displaystyle\int\frac{e^x+1}{e^x+x}dx$
$=\int\frac{1}{u}du$; $u=e^x+x$, $du=e^x+1$
$=\ln|u|+c$
$=\ln|e^x+x|+c$

(e) $=\int e^x dx+2\int dx+\int e^{-x}dx$
$=e^x+2x-e^{-x}+c$.

(f) $=\int(\ln x)^2 d(\frac{1}{2}x^2)$
$=\frac{1}{2}x^2(\ln x)^2-\underbrace{\int\frac{1}{2}x^2\cdot 2\ln x\cdot\frac{1}{x}dx}$
$\int x\ln x\,dx$
$=\int\ln x\,d\frac{1}{2}x^2$
$=\frac{1}{2}x^2\ln x-\int\frac{1}{2}x^2\cdot\frac{1}{x}dx$
$=\frac{1}{2}x^2\ln x-\frac{1}{2}\frac{1}{2}x^2+c$

$=\frac{1}{2}x^2(\ln x)^2-\frac{1}{2}x^2\ln x+\frac{1}{4}x^2+c$.

2. (10 points) Find $f(x)$ whose graph passes through the point $(4, 6)$, and

$$f'(x) = \frac{e^{\frac{2}{x}}}{x^2}.$$

$f(x) = \int f'(x)\,dx = \int \frac{1}{x^2} e^{\frac{2}{x}} dx \qquad u = \frac{2}{x} = 2x^{-1}$

$du = -2x^{-2} dx$

$= \int e^{u} (-\frac{1}{2}) du$

$= -\frac{1}{2} e^{u} + c = -\frac{1}{2} e^{\frac{2}{x}} + c.$

$f(4) \stackrel{x=4}{=} -\frac{1}{2} e^{\frac{2}{4}} + c$

$= -\frac{1}{2}\sqrt{e} + c = 6$

$\Rightarrow c = 6 + \frac{1}{2}\sqrt{e}$

$\therefore f(x) = -\frac{1}{2} e^{\frac{2}{x}} + 6 + \frac{1}{2}\sqrt{e}$

3. (10 points) Find a function f that satisfies the differential equation and the initial conditions: $f''(x) = 2$, $f'(2) = 5$, $f(2) = 10$.

$f'(x) = \int f''(x)\,dx = \int 2\,dx = 2x + c$

$f'(2) = 4 + c = 5 \Rightarrow c = 1. \quad$ i.e. $f'(x) = 2x + 1$

$f(x) = \int f'(x)\,dx = \int 2x + 1\,dx = x^2 + x + c$

$f(2) = 4 + 2 + c = 10 \Rightarrow c = 4$

$\therefore f(x) = x^2 + x + 4$

4. (20 points) Find

$$(a) \int \frac{\sin x}{1 + \cos x}\,dx \qquad (b) \int [\sec(\frac{x}{4})]^6 \tan(\frac{x}{4})\,dx$$

(a) $u = 1 + \cos x. \quad du = -\sin x\,dx$

$\int \frac{\sin x}{1 + \cos x} dx = -\int \frac{1}{u} du = -\ln|u| + c$

$= -\ln|1 + \cos x| + c.$

(b) $u = \sec(\frac{x}{4}). \quad du = \sec(\frac{x}{4}) \cdot \tan(\frac{x}{4}) \cdot \frac{1}{4} dx$

$\int [\sec(\frac{x}{4})]^6 \tan(\frac{x}{4})\,dx$

$= 4\int [\sec(\frac{x}{4})]^5 \cdot \sec(\frac{x}{4}) \cdot \tan(\frac{x}{4}) \frac{1}{4} dx$

$= 4\int u^5\,du = \frac{4}{6} u^6 + C = \frac{2}{3}[\sec(\frac{x}{4})]^6 + C.$

For your information:

- $\int \sin u\,du = -\cos u + C$
- $\int \cos u\,du = \sin u + C$
- $\int \sec^2 u\,du = \tan u + C$
- $\int \sec u \tan u\,du = \sec u + C$
- $\sin^2 u + \cos^2 u = 1$ and $\tan^2 u + 1 = \sec^2 u$