12/10 商用微積分小考解答.

榮譽第一
國立東華大學 應用數學系 ____學年度第____學期 考試科目：________ □期中□期末考試試卷

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※1.

(a) $\int \frac{2e^x}{1+e^x}\,dx.$

let $u = e^x$

$du = e^x dx \Rightarrow dx = \frac{1}{u}du$

then $\int \frac{2e^x}{1+e^x}dx = \int \frac{2\cdot u}{1+u}\times\frac{1}{u}du$

$= 2\cdot\int\frac{1}{1+u}du.$

$= 2\cdot \ln|1+u| + C$

$= 2\cdot \ln|1+e^x| + C$ ※

(b) $\int \frac{x+1}{\sqrt{x}-1}dx$

let $u = \sqrt{x}-1$ $\left(x = (u+1)^2 \right)$

$du = \frac{1}{2\sqrt{x}}dx \Rightarrow dx = 2\cdot(u+1)\,du$

then $\int \frac{x+1}{\sqrt{x}-1}dx = \int \frac{(u+1)^2+1}{(u+1)-1}\times 2\cdot(u+1)du$

$= \int \frac{2u^3+6u^2+8u+4}{u}du.$

$= 2\times\int u^2du + 6\times\int u\,du$

$+ 8\times\int 1\,du + 4\times\int\frac{1}{u}du$

$= 2\times(\frac{1}{3}u^3+C_1) + 6\times(\frac{1}{2}u^2+C_2)$

$+ 8\times(u+C_3) + 4\times(\ln|u|+C_4)$

$= \frac{2}{3}u^3 + 3u^2 + 8u + 4\ln u + C$

$= \frac{2}{3}(\sqrt{x}-1)^3 + 3(\sqrt{x}-1)^2 + 8(\sqrt{x}-1)$

$+ 4\ln|\sqrt{x}-1| + C$ ※

(c) $\int xe^{x^2} - \frac{x}{x^2+2}\,dx.$

let $u = x^2$ $\left(x = \sqrt{u} \right)$

$du = 2x\,dx \Rightarrow dx = \frac{1}{2\sqrt{u}}du.$

then $\int xe^{x^2} - \frac{x}{x^2+2}dx$

$= \int \left(\sqrt{u}e^u - \frac{\sqrt{u}}{u+2}\right)\times\frac{1}{2\sqrt{u}}du$

$= \frac{1}{2}\int e^u - \frac{1}{u+2}du$

$= \frac{1}{2}\left[\int e^u du - \int\frac{1}{u+2}du\right]$

$= \frac{1}{2}\left[(e^u+C_1) - (\ln|u+2|+C_2)\right]$

$= \frac{1}{2}(e^u - \ln|u+2|) + C$

$= \frac{1}{2}(e^{x^2} - \ln|x^2+2|) + C$ ※

(d) $\int \frac{3e^{2x}}{(1+3e^x)^2}dx$ let $u = 1+3e^x$

$du = 3e^x dx$

$= \int \frac{u-1}{3u^2}du$ $\left(e^x = \frac{u-1}{3} \right)$

$= \frac{1}{3}\left[\int\frac{1}{u}du - \int\frac{1}{u^2}du\right]$

$= \frac{1}{3}\left[\ln|u| + \frac{1}{u}\right] + C$

$= \frac{1}{3}\ln(1+3e^x) + \frac{1}{3(1+3e^x)} + C$

※

(e) $\int \frac{1}{(9-x^2)^{\frac{3}{2}}}\,dx$

let $x = 3\sin\theta \qquad dx = 3\cos\theta\,d\theta$

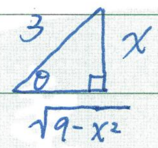
$\Rightarrow \int \frac{1}{(9-9\sin^2\theta)^{\frac{3}{2}}} \cdot 3\cos\theta\,d\theta$

$= \int \frac{3\cos\theta}{27\cos^3\theta}\,d\theta$

$= \frac{1}{9}\int \frac{1}{\cos^2\theta}\,d\theta$

$= \frac{1}{9}\int \sec^2\theta\,d\theta$

$= \frac{1}{9}\tan\theta + C$



$= \frac{x}{9\sqrt{9-x^2}} + C$ ※

(f) $\int (\ln x)^3\,dx \qquad u=(\ln x)^3,\ dv=dx$

$du = 3(\ln x)^2\frac{1}{x}dx,\ v=x$

$= x(\ln x)^3 - \int x\cdot 3(\ln x)^2\frac{1}{x}\,dx$

$= x(\ln x)^3 - 3\int (\ln x)^2\,dx$

$u=(\ln x)^2,\ dv=dx$

$du = 2(\ln x)\cdot\frac{1}{x}dx,\ v=x$

$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int \underset{u}{\ln x}\,\underset{dv}{dx}\right]$

$= x(\ln x)^3 - 3x(\ln x)^2 + 6\left[x(\ln x) - \int 1\,dx\right]$

$= x(\ln x)^3 - 3x(\ln x)^2 + 6x(\ln x) - 6x + C$ ※

※2.

(a). $y' = \frac{y\ln x}{x} \Rightarrow \frac{dy}{dx} = y\cdot\frac{\ln x}{x}$

$\Rightarrow \int \frac{1}{y}\,dy = \int \frac{\ln x}{x}\,dx$

(Right Hand Sided).

For RHS:

let $u = \ln x \quad (\!(x = e^u)\!)$

$du = \frac{1}{x}dx \Rightarrow dx = e^u\,du$

⇒ conti. ※2. (a).

$\int \frac{1}{y}\,dy = \int \frac{\ln x}{x}\,dx$

$\Rightarrow \ln|y| + C_1 = \int \frac{u}{e^u} e^u\,du$

$= \frac{1}{2}u^2 + C_2$

$= \frac{1}{2}(\ln x)^2 + C_2$

$\Rightarrow \ln|y| = \frac{1}{2}(\ln x)^2 + (C_2 - C_1)$

取e, let $C = C_2 - C_1$ $\Rightarrow y = e^{\frac{1}{2}(\ln x)^2 + C}$

$= e^C \times e^{\frac{1}{2}(\ln x)^2}$

let $e^C = C'$

$= C' e^{\frac{1}{2}(\ln x)^2}$ ※

(b). $y' = xye^x \Rightarrow \frac{dy}{dx} = y\cdot xe^x$

$\Rightarrow \int \frac{1}{y}\,dy = \int xe^x\,dx$

For RHS:

let $u = x,\ dv = e^x dx$

then $du = dx,\ v = e^x$

$\int \frac{1}{y}\,dy = \int xe^x\,dx$

$\Rightarrow \ln|y| + C_1 = \left[xe^x - \int e^x\,dx\right] + C_2$

$= xe^x - (e^x + C_3) + C_2$

$\Rightarrow \ln|y| = (x-1)e^x + (C_2 - C_1 - C_3)$

取e, let $C = C_2 - C_1 - C_3$ $\Rightarrow y = e^{(x-1)e^x + C}$

$= e^C \times e^{(x-1)e^x}$

let $C' = e^C$

$= C' \times e^{(x-1)e^x}$

And given $y(1) = 1$

$\Rightarrow y(1) = C' \times e^{(1-1)e^1}$

$= C' \times e^0 = C' \times 1 = 1$

⇒ conti. Hence $C' = 1$, $y = e^{(x-1)e^x}$ ※

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※3：

(a) $\int \sin(\ln x)\,dx$

let $u=\sin(\ln x)$, $dv=dx$

$du=\cos(\ln x)\cdot\frac{1}{x}dx$, $v=x$

$\int \sin(\ln x)dx = x\cdot\sin(\ln x)-\int x\cos(\ln x)\cdot\frac{1}{x}dx$

$=x\sin(\ln x)-\int \underbrace{\cos(\ln x)}_{u}\,\underbrace{dx}_{dv}$

$=x\sin(\ln x)-[x\cos(\ln x)+\int \sin(\ln x)dx]$

$=x\sin(\ln x)-x\cos(\ln x)-\int \sin(\ln x)dx$

Let $I=\int \sin(\ln x)dx$

such that

$I=x\sin(\ln x)-x\cos(\ln x)-I$

$\Rightarrow 2I = x\sin(\ln x)-x\cos(\ln x)$

$\Rightarrow \int \sin(\ln x)dx = \frac{x\sin(\ln x)-x\cos(\ln x)}{2}+C$

※

(b) $\int \sec(\pi x)\,dx$

$=\int \sec(\pi x)\cdot\frac{\sec(\pi x)+\tan(\pi x)}{\sec(\pi x)+\tan(\pi x)}dx$

$=\int \frac{\sec^2(\pi x)+\sec(\pi x)\tan(\pi x)}{\sec(\pi x)+\tan(\pi x)}dx$ ㊀

let $u=\sec(\pi x)+\tan(\pi x)$

$du=\pi(\sec(\pi x)\tan(\pi x)+\sec^2(\pi x))dx$

㊀ $=\frac{1}{\pi}\int \frac{1}{u}du$

$=\frac{1}{\pi}\ln|u|+C$

$=\frac{1}{\pi}\ln|\sec(\pi x)+\tan(\pi x)|+C$

※