榮譽第一 11/05 商用微積分解答.

國立東華大學 應用數學系 ______學年度第____學期 考試科目：______________ □期中□期末考試試卷

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※1

$$f'(x) = 2x(2-x) + (x^2+1)\cdot(-1)$$
$$= 4x - 2x^2 - x^2 - 1$$
$$= -3x^2 + 4x - 1 \overset{\text{Set}}{=} 0$$
$$\Rightarrow (-3x+1)(x-1) = 0$$
$$\Rightarrow x = 1 \text{ or } \frac{1}{3}$$

And $f(1) = 2$, $f(\frac{1}{3}) = \frac{50}{27}$

Hence, the points are

$(1, 2)$, $(\frac{1}{3}, \frac{50}{27})$.

※2 (1)

$$f'(x) = (2x^2+7)^{\frac{1}{2}} + x\cdot\frac{1}{2}\cdot(2x^2+7)^{-\frac{1}{2}}\cdot 4x$$
$$= (2x^2+7)^{\frac{1}{2}} + 2x^2(2x^2+7)^{-\frac{1}{2}}$$
$$\Rightarrow f'(3) = (2\cdot 9+7)^{\frac{1}{2}} + 2\cdot 9\cdot(2\cdot 9+7)^{-\frac{1}{2}}$$
$$= 5 + 18\cdot\frac{1}{5}$$
$$= \frac{43}{5}$$

Thus, the targent line is

$$(y-15) = \frac{43}{5}(x-3)$$
$$\Rightarrow y = \frac{43}{5}x - \frac{54}{5}.$$

(2)

$$\frac{d}{dx}(x^2y^3 - y^2 + xy - 1) = \frac{d}{dx}(0)$$
$$\Rightarrow 2xy^3 + x^2\cdot 3y^2\cdot\frac{dy}{dx} - 2y\cdot\frac{dy}{dx} + y + x\cdot\frac{dy}{dx} = 0$$
$$\Rightarrow (3x^2y^2 - 2y + x)\frac{dy}{dx} = -2xy^3 - y$$
$$\Rightarrow \frac{dy}{dx} = \frac{-2xy^3 - y}{3x^2y^2 - 2y + x}$$

$\Rightarrow$ 斜率 $\left.\frac{dy}{dx}\right|_{(x,y)=(1,1)} = \frac{-2-1}{3-2+1} = -\frac{3}{2}$

Thus, the targent line is

$$(y-1) = -\frac{3}{2}(x-1)$$
$$\Rightarrow y = -\frac{3}{2}x + \frac{5}{2}$$

※3. (1).

$$F'(x) = g'(f(x))\cdot f'(x).$$
$$\Rightarrow F'(2) = g'(f(2))\,f'(2)$$
$$= g'(3)\cdot(-3)$$
$$= 4\cdot(-3) = -12 \text{ ※}.$$

(2). $y = \tan(2x^2)$

(i). $$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin(2x^2)}{\cos(2x^2)}\right)$$
$$= \frac{\cos(2x^2)(4x)\cos(2x^2) - \sin(2x^2)(-\sin(2x^2))(4x)}{[\cos(2x^2)]^2}$$
$$= \frac{4x}{\cos^2(2x^2)} = 4x\cdot\sec^2(2x^2) \text{ ※}.$$

(ii) $$\frac{dy}{dx} = \frac{d}{dx}\tan(2x^2)$$
$$= \sec^2(2x^2) \text{ ※}$$

(3). $y^2 - xy = 8$.

$$\frac{d}{dx}(y^2 - xy) = \frac{d}{dx}(8).$$
$$\Rightarrow 2y\frac{dy}{dx} - [y + x\frac{dy}{dx}] = 0.$$
$$\Rightarrow \frac{dy}{dx} = \frac{y}{2y-x}$$

⇒ conti.

⇒ conti. #3 (3).

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{y}{2y-x}\right)$$

$$= \frac{\left(1\cdot\frac{dy}{dx}\right)(2y-x) - y\cdot\left(2\frac{dy}{dx}-1\right)}{(2y-x)^2}$$

$$\left(代入\ \frac{dy}{dx} = \frac{y}{2y-x}\right) = \frac{\left(\frac{y}{2y-x}\right)(2y-x) - y\cdot\left(2\cdot\frac{y}{2y-x}-1\right)}{(2y-x)^2}$$

$$= \frac{2y(y-x)}{(2y-x)^3} \quad ※.$$

※4 Let $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

$$dy = f'(x)\,dx$$

$$= \left(\tfrac{1}{2}x^{-\frac{1}{2}} - \tfrac{1}{2}x^{-\frac{3}{2}}\right)dx$$

Here, $x = 4$ and $dx = 4.02 - 4 = 0.02$

$$\Rightarrow dy = \left(\tfrac{1}{4} - \tfrac{1}{16}\right)\cdot 0.02$$

$$= \frac{3}{16}\cdot\frac{1}{50} \approx \Delta y$$

Thus, $f(4.02) = f(4) + \Delta y$

$$\approx f(4) + dy$$

$$= 2 + \tfrac{1}{2} + \frac{3}{16\cdot 50}$$

$$= 2.50375$$

※5.

$$f(x) = 2x^3 - 3x^2 - 16x + 3.$$

$$f'(x) = 6x^2 - 6x - 16$$

to find. $f'(x) = -4$.

$$\Rightarrow 6x^2 - 6x - 16 = -4.$$

$$\Rightarrow 6x^2 - 6x - 12 = 0$$

$$\Rightarrow 6(x-2)(x+1) = 0.$$

$$\Rightarrow x = 2 \text{ or } -1.$$

And $f(2) = -25$, $f(-1) = 14$.

Hence the points are.

$(2, -25)$ and $(-1, 14)$ ※.

※6

$$f(x) = \frac{x^2}{x-1}.$$

$$f'(x) = \frac{2x(x-1) - x^2\cdot 1}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$$f''(x) = \frac{d}{dx}\left(\frac{x^2-2x}{(x-1)^2}\right)$$

$$= \frac{(2x-2)(x-1)^2 - (x^2-2x)\cdot 2\cdot(x-1)}{(x-1)^4}$$

$$= \frac{2}{(x-1)^3}$$

(a). Setting $f'(x) = 0$.

$\Rightarrow x = 0$ or 2.

+++ 0 ---- 2 +++

① Since $x < 0 \Rightarrow f'(x) > 0$.

② Since $0 < x < 2 \Rightarrow f'(x) < 0$.

③ Since $x > 2 \Rightarrow f'(x) > 0$

Hence f is increasing on $(-\infty, 0)$ and $(2, \infty)$

and decreasing on $(0, 2)$, $x \neq 1$ ※

(b) Setting $f'(x) = 0$

$\Rightarrow x = 0$ or 2.

$\Rightarrow f(0) = 0$, $f(2) = 4$.

and according $f''(x)$.

$f''(0) = -2 < 0 \Rightarrow$ relative max

$f''(2) = 2 > 0 \Rightarrow$ relative min

Hence we have relative max at $(0, 0)$

and relative min at $(2, 4)$ ※.

(c). According $f''(x)$.

$f''(x)$ is not defined at $x = 1$.

and $f''(x) < 0$ on $(-\infty, 1)$

$f''(x) > 0$ on $(1, \infty)$

Hence f concave downward on $(-\infty, 1)$

upward on $(1, \infty)$ ※

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(d). Since

$$f''(x) = \frac{2}{(x-1)^3}$$

we can't found an x

s.t. $f''(x) = 0$.

Hence the inflection p.t.

is not exist.

※7

$$f'(s) = (1-s^2)^{\frac{1}{2}} + s \cdot \frac{1}{2} \cdot (1-s^2)^{-\frac{1}{2}} \cdot (-2s)$$

$$= (1-s^2)^{\frac{1}{2}} - s^2 (1-s^2)^{-\frac{1}{2}} \overset{\text{Set}}{=} 0$$

$$\Rightarrow (1-s^2)^{\frac{1}{2}} = s^2 (1-s^2)^{-\frac{1}{2}}$$

$$\Rightarrow (1-s^2) = s^2$$

$$\Rightarrow 2s^2 = 1$$

$$\Rightarrow s = \pm\sqrt{\tfrac{1}{2}}$$

$\because$ $f(\sqrt{\tfrac{1}{2}}) = \sqrt{\tfrac{1}{2}} \cdot \sqrt{1-\tfrac{1}{2}} = \frac{1}{2}$,

$f(-\sqrt{\tfrac{1}{2}}) = -\sqrt{\tfrac{1}{2}} \cdot \sqrt{1-\tfrac{1}{2}} = \frac{-1}{2}$,

$f(1) = 1 \cdot \sqrt{1-1} = 0 = f(-1)$

$\therefore$ Absolute maximum is $x = \sqrt{\tfrac{1}{2}}$, $f(\sqrt{\tfrac{1}{2}}) = \frac{1}{2}$

Absolute minimum is $x = -\sqrt{\tfrac{1}{2}}$, $f(-\sqrt{\tfrac{1}{2}}) = \frac{-1}{2}$