1. Let $X_1, \ldots, X_n$ be a random sample from a $U(0, \theta)$, with unknown parameter $\theta > 0$, and let $Y = X_{(n)} = \max\{X_1, \ldots, X_n\}$.
   (a) Let $0 \le c < d$ be given constant, and consider the random interval $C_1(Y) = [Y + c, Y + d]$. Calculate the coverage probability function of $C_1(Y)$, then show that the confidence coefficient of $C_1(Y)$ is 0.
   (b) Let $1 \le a < b$ be given constant, and consider the random interval $C_2(Y) = [aY, bY]$. Show that the coverage probability function of $C_2(Y)$ is a constant, hence equals to the confidence coefficient of $C_2(Y)$.
   (c) Now, for a given $\alpha \in (0, 1)$, give three different pairs of a and b in (b) such that the confidence coefficient of $C_2(Y)$ is $1 - \alpha$.

2. Assume that α_1 and α_2 are any numbers such that $0 \le \alpha_1 \le \alpha_2 \le 1$. Let U_α be an exact upper confidence bound for θ, i.e. $P_\theta(\theta \le U_\alpha(X)) = \alpha$, $\forall\, \theta$. If $U_{\alpha_1}(x) \le U_{\alpha_2}(x)$, $\forall\, x$, prove that for all θ, $P_\theta\{\theta \in [U_{\alpha_1}, U_{\alpha_2}]\} \ge \alpha_2 - \alpha_1$.

3. Let $X_1, \ldots, X_n$ be *i.i.d.* from $N(\mu, \sigma_0^2)$, with $\sigma_0 > 0$ and $\alpha \in (0, 1)$ are given constants.
   (a) Consider the random lower confidence limit for θ:
   $$C_1(X) = \left[\bar{X} - \frac{\sigma_0}{\sqrt{n}} Z_\alpha, \infty\right).$$
   Calculate the coverage probabilities and the (relevant) false coverage probabilities of $C_1(X)$. Is $C_1(X)$ an unbiased confidence set for θ?
   (b) Also, consider the random upper confidence limit for θ:
   $$C_2(X) = \left(-\infty, \bar{X} + \frac{\sigma_0}{\sqrt{n}} Z_\alpha\right].$$
   Calculate the coverage probabilities and the (relevant) false coverage probabilities of $C_2(X)$. Is $C_2(X)$ an unbiased confidence set for θ?
   (c) Now, consider the random confidence interval for θ:
   $$C_3(X) = \left[\bar{X} - \frac{\sigma_0}{\sqrt{n}} Z_{\alpha/2}, \bar{X} + \frac{\sigma_0}{\sqrt{n}} Z_{\alpha/2}\right].$$
   Calculate the coverage probabilities and the (relevant) false coverage probabilities of $C_3(X)$. Is $C_3(X)$ an unbiased confidence set for θ?

4. Find a $1 - \alpha$ confidence interval for θ, given $X_1, \ldots, X_n$ i.i.d. with p.d.f.
   (a) $f(x; \theta) = 1$, $\theta - 0.5 < x < \theta + 0.5$; 0, otherwise.
   (b) $f(x; \theta) = 2x/\theta^2$, $(\theta > 0)$, $0 < x < \theta$; 0, otherwise.

5. Let $X_1, \dots, X_n$ be a random sample from a $N(0, \sigma_x^2)$ and $Y_1, \dots, Y_m$ be a random sample from $N(0, \sigma_y^2)$, independent of the $X's$. Define $\lambda = \sigma_y^2/\sigma_x^2$. Find a $1-\alpha$ confidence interval for λ by inverting that LRT.

6. Let $X_1, \dots, X_n$ be i.i.d. random variables from $N(\theta, \sigma^2)$, where σ^2 is also unknown. As usual, $\bar{X} = \sum X_i/n$ and $S^2 = \sum (X_i - \bar{X})^2/(n-1)$.
   (a) Show that the interval $\{\theta : \theta \le \bar{X} + S t_{n-1,\alpha}/\sqrt{n}\} = (-\infty, \bar{X} + S\, t_{n-1,\alpha}/\sqrt{n}]$ can be derived by inverting the acceptance region of an LRT.
   (b) Show that the interval $[\bar{X} \pm S\, t_{n-1,\alpha/2}/\sqrt{n}]$ can also be derived by inverting the acceptance region of an LRT.
   (c) Show that the intervals in parts (a) and (b) are unbiased.

7. Let $X_1, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$.
   (a) If σ^2 is known, find a minimum sample size n to quarantee that the 95% two-sided UMAU confidence interval for μ will have length no more than $\sigma/4$.
   (b) If σ^2 is unknown, how to find a minimum sample size n to quarantee, with probability 0.9, that the 95% two-sided UMAU confidence interval for μ will have length no more than $\sigma/4$?

8. Let a random variable $X \sim f(x;\theta)$, where f is a p.d.f. defined as

$$f(x;\theta) = \frac{e^{(x-\theta)}}{(1+e^{(x-\theta)})^2}, \quad x \in R,\ \theta \in R.$$

   Based on one observation, X, find the UMA one-sided $1-\alpha$ confidence interval of the form $\{\theta : \theta \le U(X)\}$.