

1. Suppose that $X_1, \dots, X_n$ are *i.i.d.* r.v.'s from $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Given μ_0 and $0 < \alpha < 1$. Derive the level α LRT (likelihood ratio test) for testing $H_0 : \mu \leq \mu_0$ v.s. $H_1 : \mu > \mu_0$.
2. Let $f(x; \theta) = \theta e^{-\theta x}$, for $x > 0$, and $f(x; \theta) = 0$, otherwise. Suppose that $X_1, \dots, X_n$ are *i.i.d.* r.v.'s from the common probability density function $f(x; \theta_1)$, and $Y_1, \dots, Y_m$ *i.i.d.* r.v.'s from $f(x; \theta_2)$. Also assume that the X 's are independent of the Y 's. For testing $H_0 : \theta_1 = \theta_2$ v.s. $H_1 : \theta_1 \neq \theta_2$, given $0 < \alpha < 1$,
 - (a) show that the LRT can be based on the statistic

$$T = \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n X_i + \sum_{j=1}^m Y_j},$$

- (b) find the distribution of T when H_0 is true, then
- (c) show how the level α LRT can become an F test.

And 13.7: 1 (i, ii, iii), 3 from textbook.