1. A sample of size 1 is taken from the *p.d.f.*

$$f(x;\theta)=\begin{cases}\frac{2}{\theta^2}(\theta-x) & \text{if } 0<x<\theta,\\ 0 & \text{otherwise.}\end{cases}$$

Let θ_0, θ_1 with $\theta_0>\theta_1$ be two given constants.

(a) Find the most powerful (MP) level $\alpha=0.05$ test for testing $H_0:\theta=\theta_0$ v.s. $H_1:\theta=\theta_1$. Calculate its power.

(b) Write out the test in part (a) for the case where $\theta_0=2$ and $\theta_1=1$.

2. Let X be random variable with *p.d.f.* f whoch can be either f_0 or else f_1, where f_0 is the *p.d.f.* of $N(0,1)$ and f_1 is the *p.d.f.* of Cauchy(0, 1) (i.e. $f_1(x)=1/(\pi(1+x^2))$, $x\in R$). Find the MP level α test for testing $H_0:f=f_0$ v.s. $H_1:f=f_1$. Calculate its power.

3. Let $X_1,\ldots,X_n$ be *i.i.d.* $U(0,\theta)$ r.v.'s and $X_{(n)}=\max\{X_1,\ldots,X_n\}$. Also let θ_0, θ_1 with $0<\theta_0<\theta_1$ and $0<\alpha<1$ be given constants . For testing $H_0:\theta=\theta_0$ v.s. $H_1:\theta=\theta_1$,

(a) show that the test

$$\phi_1(x_1,\ldots,x_n)=\phi(x_{(n)})=\begin{cases}1 & \text{if } \theta_0<x_{(n)},\\ \alpha & \text{otherwise}\end{cases}$$

is a MP level α test.

(b) Define a test ϕ_2 as

$$\phi_2(x_1,\ldots,x_n)=\phi_2(x_{(n)})=\begin{cases}1 & \text{if } k\le x_{(n)},\\ 0 & \text{otherwise,}\end{cases}$$

where k is determined such that $E_{\theta_0}\phi_2(x_{(n)})=\alpha$.
Determine the value of k, and show that ϕ_2 is also a MP level α test for testing $H_0:\theta=\theta_0$ v.s. $H_1:\theta=\theta_1$.

4. Let $X_1,\ldots,X_n$ be *i.i.d.* Gamma$(1,\beta)$. Find the MP level $\alpha=0.05$ test for testing $H_0:\beta=2$ v.s. $H_1:\beta=4$. (Note that it is a Chi-squared test.)