1. Let X be a discrete random variable whose p.m.f. is for $k = 1, 2, \ldots, 7$, $P_p(X = k) = p_k$ where $p = (p_1, p_2, \ldots, p_7)$ with $p_k \geq 0$, $\forall\, k$ and $\sum p_k = 1$. Based on observing one X, give THREE different level $\alpha = 0.05$ tests for testing
   $H_0 : p = (0.01, 0.01, 0.02, 0.01, 0.01, 0.01, 0.93)$ $v.s.$
   $H_1 : p = (0.06, 0.05, 0.04, 0.03, 0.02, 0.01, 0.79)$.
   Compute the probability of Type-II error for your tests.
   Do the same with level 0.035.

2. Let $X \sim \text{Gamma}(1, \beta)$. To test $H_0 : \beta = 1$ v.s. $H_1 : \beta > 1$, suppose student A uses the non-randomized test
$$\phi_A(x) = \begin{cases} 1 & \text{if } x > c, \\ 0 & \text{if } x \leq c.; \end{cases}$$
   and B uses
$$\phi_B(x) = \begin{cases} 1 & \text{if } x < k, \\ 0 & \text{if } x \geq k.; \end{cases}$$
   Find the constants c and k so that both tests have the same size 0.05 and then derive and compare their power functions. Which test is better? Why?

3. Let $X_1, \ldots, X_n$ be *i.i.d.* $U(0, \theta)$ r.v.'s and $X_{(n)} = \max\{X_1, \ldots, X_n\}$. Also let θ_0, θ_1 with $0 < \theta_0 < \theta_1$ and $0 < \alpha < 1$ be given constants . For testing $H_0 : \theta = \theta_0$ v.s. $H_1 : \theta = \theta_1$, Consider the test defined by
$$\phi_1(x_1, \ldots, x_n) = \phi_1(x_{(n)}) = \begin{cases} 1 & \text{if } \theta_0 < x_{(n)}, \\ \alpha & \text{otherwise.} \end{cases}$$
   Show that it is a level α test and calculate its power.
   Now, define another test ϕ_2 as
$$\phi_2(x_1, \ldots, x_n) = \phi_2(x_{(n)}) = \begin{cases} 1 & \text{if } k \leq x_{(n)}, \\ 0 & \text{otherwise,} \end{cases}$$
   where k is determined such that $E_{\theta_0}[\phi_2(X_{(n)})] = \alpha$.
   Determine the value of k, and show that ϕ_2 has the same power as ϕ_1

and problem 13.1.1 of the textbook. (4 problems in total)