Note: The exam has 5 questions, for a total of 125 points.
Explain your answer and write down necessary details of your calculation.
No explanation/details = No credits. *Good Luck!*

1. *True or False?* In the following questions, determine whether the statement is true or false. Justify/Explain your answer. Give an example or explain briefly if the answer is "True", otherwise, give a counterexample.
   - (5) (a) Let f and F be the pdf (probability density function) and cdf (cumulative distribution function) of a discrete random variable then $F'(x) = f(x)$ for all $x \in (-\infty, \infty)$.
   - (5) (b) If X, Y are two random variables and g, h are two continuous functions then $E(g(X)h(Y)) = E(g(X))E(h(Y))$ provided all expectations exist.
2. Let X and Y be r.v.'s whose joint pdf f is given by $f(x, y) = cxy1_{(0,1)\times(0,1)}(x, y)$.
   - (5) (a) Determine the constant c such that $f(x, y)$ is indeed a pdf.
   - (10) (b) Compute the marginal pdf f_X and marginal cdf $F_X(x)$ of the random variable X.
   - (10) (c) Compute $P(0.5 < F_X(X) < 4)$.
   - (10) (d) Compute $P(X < 0.5, Y < 0.3)$.
   - (10) (e) Let $Z = max(X, Y)$. Compute $P(Z > 0.5)$.
   - (10) (f) Compute pdf of (T, W) where $T = X - Y, W = X + Y$.
   - (10) (g) Are T and W independent?
3. Let $X \sim U(0, 1)$ and define $Y = I_{B_1}(X) - I_{B_2}(X)$ where $I_A(x)$, the indicator function of event A, takes value 1 if $x \in A$; 0, otherwise. Let $B_1 = (0.1, 0.2), B_2 = (0.3, 0.5)$.
   - (10) (a) Determine the pdf of Y.
   - (5) (b) Verify that the pdf of Y you derived above is indeed a pdf.
4. Let $(X_1, X_2) \sim N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ where $\mu_1, \mu_2 \in R$, $\sigma_1^2, \sigma_2^2 > 0$ and $|\rho| < 1$. It is known that (you may use them as given)
$$X_2|X_1 = x_1 \sim N(b(x_1), \sigma_2^2(1 - \rho^2))$$
with $b(x_1) = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x_1 - \mu_1)$ and marginally $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$.
   - (10) (a) Derive $Var(E(X_2|X_1))$.
   - (10) (b) Calculate $E(X_1 - X_2)^2$.
   - (10) (c) Assume it is known that $\sigma_1 = \sigma_2 = 1$. Find a number $1 > c > 0$ such that $P(|X_1 - \mu_1| > 2) \leq c$. **Hint**: Chebyshev's inequality.
5. (5) Estimate the points you will get in the exam (excluding this problem).