Linear Methods for Regression

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Statistical Decision Theory: Versions of Expected Losses

[Point Estimation Problem]
Let $X\_, \cdots, X_n \sim f_\theta$, for example, pdf of $N(\theta, \sigma^2)$ or pmf of $Bernoulli(\theta)$ ObjectIFind $\hat{\theta}_*$

Complete Class Theorem

- A class $\mathcal{C}$ is *complete* if for any decision δ not in $\mathcal{C}$, there exists a decision δ' which dominates δ.
- Under some regularity conditions, the class of eneralized Bayes procedures form a complete class.
- Implication: Search no further. ork with eneralized Bayes procedures.

$(Y| \quad)$

Consider $X \in R^p$, $Y \in R$ with joint prob distribution $P_{Y,X}$. Seek a ftn f for predicting Y given X. Under $L(Y, f(X)) \quad (Y - f(X))^2$, *squaed error loss* , in the spirit of Bayes risk, find f min

$$EPE(f) \quad E(Y - f(X))^2 \quad (y \ \ f(x))^2 dP_{Y,X}(y, x) \qquad (1)$$

KNN as $(Y| \ )$

Let $T \quad (Y_i, X_i)_i^n$ _ be the training data.

- $\hat{f}(x) \quad Ave(y_i | x_i \in N_k(x))$, where "Ave" average, $N_k(x)$ is the neighborhood containing the k points in T closest to x.
- expectation is approximated by averagig over sample space .
- conditioning at a point is relaxed to conditionig on some region "clos

KNN as $(Y|)$

- $\hat{f}(x)$ $Ave(y_i|x_i \in N_k(x))$, whee "Ave" average, $N_k(x)$ is the neghboghd cn523(t)-1.aining k the points in T closest to x.
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How good is NN Ratonale Serc is over

- Sample sizesually small
- As p inceases, $N_k(x)$ becmes huge
- Convegec
  - Cverge holds. $\hat{f} \to f$ as $n \to \infty$
  - Slower rate of convergence.

LS as ($Y|$)

- $f(x) \approx x^T \beta$
- Plug this f int1,2.97773TR10310.9091Tf74.070Td[(S)4.6255]TER40

KNN vs. LS

- LS assumes $f(x) \approx$ globally by a linear function
- NN assumes $f(x) \approx$ locally by a const function
- Functional approximation