Note: The exam has 5 questions, for a total of 120 points. Explain your answer and write down the necessary details of your calculation. No explanation/details = No credits. *Good Luck!*

Some definitions and function values for your reference:

- $\Phi(x) = P[Z \leq x] = \int_{-\infty}^{x} \varphi(t)dt$, cdf of a standard normal random variable $Z \sim N(0, 1)$.
- $\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, pdf of a standard normal r.v.
- $\Phi(1.96) = 0.975$, $\Phi(1.645) = 0.95$, $\Phi(1.282) = .90$.

1. We are interested in a quality characteristic which can be assessing through a $\bar{x}$ chart for a normal mean. If the mean shifts from the in-control value–say, μ_0–to another value $\mu_1 = \mu_0 + k\sigma$. Recall that $\bar{X} \sim N(\mu, \sigma^2)$. The control limits are UCL=$\mu_0 + L\sigma/\sqrt{n}$ and LCL=$\mu_0 - L\sigma/\sqrt{n}$

(10) (a) Find β-risk, i.e. the probability of *not* detecting this shift (from μ_0 to μ_1) on the first subsequent sample.

(10) (b) Find the ARL (Average Run Length) before this shift is detected.

(10) (c) Does smaller ARL indicate a better control chart? Comment and discuss briefly.

(10) 2. An $\bar{x}$ chart is used to control the mean of a normally distributed quality characteristic. It is known that $\sigma = 6.0$ and $n = 4$. The center line=200, UCL=209, LCL=191. If the process mean shifts to 188, find the probability that this shift is detected on the first subsequent sample?

(20) 3. In many process control problems, we can sometimes transform a quality characteristic of variable into one of attribute. Give 2 examples such that one is appropriate and the other is inappropriate. Discuss and comment briefly.

Let $J(F) \equiv E \quad [e$